JOHN CURTIN
College of the Arts
COLLEGE OF THE ARTS

YEAR 11 MATHEMATICS SPECIALIST

TEST 2, 2018

(Vectors and Geometric proofs)

CALCULATOR-ASSUMED

Student's Name: Solutions

Total Marks: 42
Time Allowed: 50 mins

MATERIAL REQUIRED/RECOMMENDED FOR THIS TEST

Standard Items: Pens, pencils, eraser, ruler

Special Items: Up to three approved calculators
One page (unfolded A4 sheet) front and back of Notes
WACE Formula Sheet

INSTRUCTIONS TO STUDENTS

Do not open this paper until instructed to do so.
You are required to answer ALL questions.
Write answers in the spaces provided beneath each question.
Marks are shown with the questions.

Show all working clearly, in sufficient detail to allow your answers to be checked and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks.

It is recommended that students **do not use pencil**, except in diagrams.

Question 1 **[11 marks = 1, 2, 2, 3, 3]**

If $\mathbf{a} = 5\mathbf{i} + 12\mathbf{j}$, $\mathbf{b} = 3\mathbf{i} - 4\mathbf{j}$ and $\mathbf{c} = x\mathbf{i} - \mathbf{j}$ find:

a) a vector in the same direction as ***a*** but twice the magnitude of ***a***.

$V = 10i + 24j$ ✓

b) a unit vector in the same direction as ***b***.

$\hat{b} = \frac{b}{|b|} = \frac{3i - 4j}{5} = \frac{3}{5}i - \frac{4}{5}j$ ✓ ✓

c) a vector in the same direction as ***b*** but the same magnitude as ***a***.

$|a| = \sqrt{5^2 + 12^2} = 13$ ✓

$V = \hat{b} \cdot |a| = \left(\frac{3}{5}i - \frac{4}{5}j\right) \times 13 = \frac{39}{5}i - \frac{52}{5}j$ ✓

d) if $|\mathbf{c}| = |\mathbf{a}|$, find the possible values of x

$|c| = 13 = \sqrt{x^2 + 1^2}$ ✓ $\therefore x^2 + 1 = 169 \quad x^2 = 168$

$\therefore x = \pm\sqrt{168} = 2\sqrt{42}$ or $-2\sqrt{42}$ ✓ ✓

$\sim \pm 12.96$

e) If $\mathbf{a} + \mathbf{b} - \mathbf{c} = -3\mathbf{i} + y\mathbf{j}$, find the values for x and y.

$a + b - c = (5 + 3 - x)i + (12 - 4 - (-1))j$ ✓

$= (8 - x)i + (9)j$

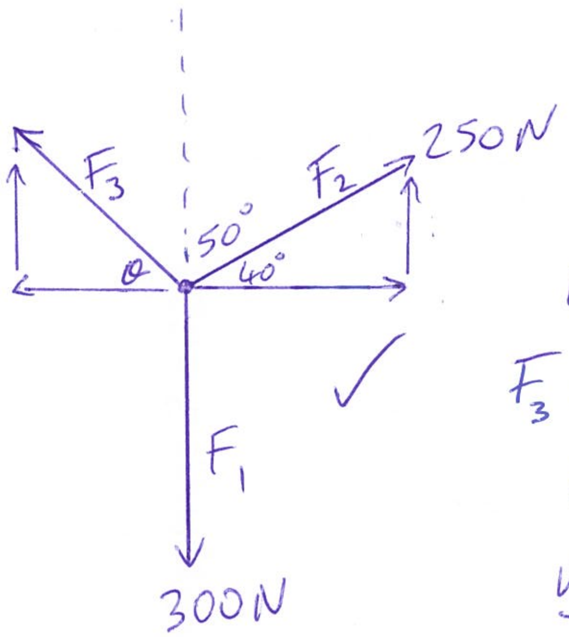
$= -3i + yj$

$\therefore \begin{cases} 8 - x = -3 \\ 9 = y \end{cases} \quad \therefore \begin{cases} x = 11 \\ y = 9 \end{cases}$ ✓ ✓

11

Question 2 [5 marks]

Three forces are applied to a body. One has magnitude 300 N and acts due south. Another has magnitude 250 N and acts on a bearing of 050º. If all three forces are in equilibrium, determine the magnitude and direction of the third force.



$F_1 = -300\underline{j}$

$F_2 = 250\cos 40\,\underline{i} + 250\sin 40\,\underline{j}$

$F_3 = -|F_3|\cos\theta\,\underline{i} + |F_3|\sin\theta\,\underline{j}$

✓

In Equilibrium:

F_3 will be:

<u>Horizontal</u>: $-250\cos 40 = -191.51\,N$ ✓

<u>Vertical</u>: $300 - 250\sin 40 = 139.30\,N$ ✓

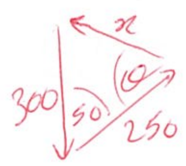
$|F_3| = \sqrt{(191.51)^2 + (139.30)^2} = \underline{236.8N}$ ✓

$\theta = \tan^{-1}\left(\frac{139.30}{191.51}\right)$

$= 36°$

Bearing $= 270 + 36 = \underline{306°}$ ✓

<u>Alternative method</u>



or $x^2 = 250^2 + 300^2 - 2(250)(300)\cos 50$

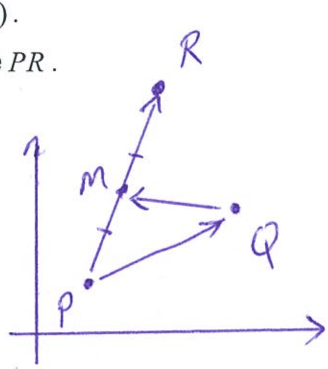
$x^2 = 56081$

$x = 236.81$

5

Question 3 [8 marks = 4, 4]

(a) A triangle *PQR* has vertices $P(1,\ 1)$, $Q(5,\ 3)$ and $R(3,\ 7)$.
Determine the vector $\overrightarrow{QM}$ where *M* is the midpoint of side PR.



$\overrightarrow{PR} = 2\underline{i} + 6\underline{j}$

$\overrightarrow{PQ} = 4\underline{i} + 2\underline{j}$

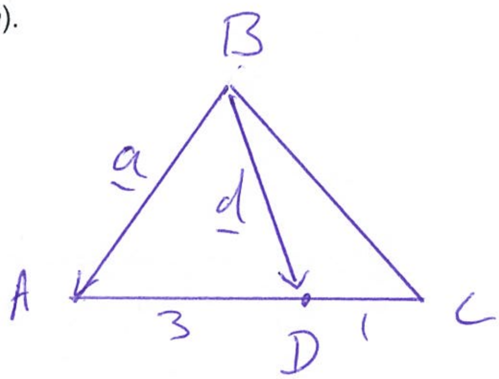
$\overrightarrow{PM} = \frac{1}{2}(\overrightarrow{PR}) = 1\underline{i} + 3\underline{j}$

$\overrightarrow{QM} = \overrightarrow{PM} - \overrightarrow{PQ} = (i+3j) - (4i+2j)$

$= -3i + j$

(b) *ABC* is a triangle with point *D* on side $\overrightarrow{AC}$ such that $\overrightarrow{AD} = \frac{3}{4}\overrightarrow{AC}$.

If $\overrightarrow{BA} = \boldsymbol{a}$ and $\overrightarrow{BD} = \mathbf{d}$, show that $\overrightarrow{BC} = \frac{1}{3}(4\boldsymbol{d} - \boldsymbol{a})$.



$\overrightarrow{AD} = \underline{d} - \underline{a}$

$\overrightarrow{DC} = \frac{1}{3}AD = \frac{1}{3}(\underline{d} - \underline{a})$

$\therefore BC = \overrightarrow{BD} + \overrightarrow{DC}$

$= d + \frac{1}{3}(d-a)$

$= \frac{4}{3}\underline{d} - \frac{1}{3}\underline{a}$

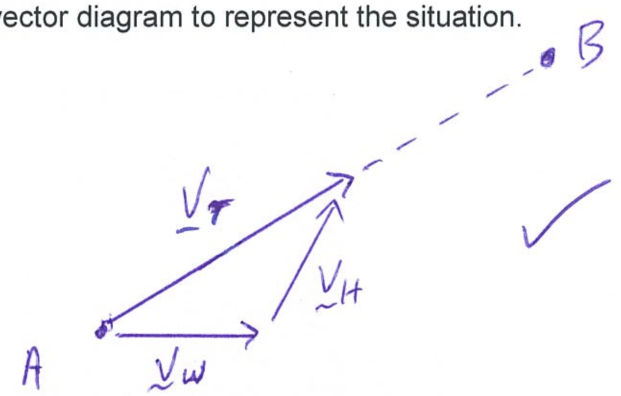
$= \frac{1}{3}(4\underline{d} - \underline{a})$

8

Question 4 [~~8~~ 10 marks = 1, 5, ~~1, 1~~ 2, 2]

A helicopter capable of flying at a speed of 39km/hr in still air, takes off from airport A to airport B such that $\overrightarrow{AB} = 300\mathbf{i} + 100\mathbf{j}$. Throughout the journey, the helicopter encounters a wind blowing with velocity of $9\mathbf{i}$ km/hr.

(a) Draw vector diagram to represent the situation.



(b) Find the velocity vector, in the form a**i** + b**j**, the pilot should set so that the helicopter flies directly to airport B.

let $\underline{V}_H = a\underline{i} + b\underline{j}$

$\underline{V}_w + \underline{V}_H = \lambda(300i + 100j)$ ✓

$9i + ai + bj = 300\lambda i + 100\lambda j$

$\left.\begin{aligned} a + 9 &= 300\lambda \\ b &= 100\lambda \end{aligned}\right\}$ ✓

$\frac{a+9}{b} = \frac{300}{100} = 3$

$a + 9 = 3b$

$a = 3b - 9$ ✓

$|a\underline{i} + b\underline{j}| = 39$

$a^2 + b^2 = 39^2$

$(3b-9)^2 - b^2 = 39^2$ ✓

$b = 15$ or $-\frac{48}{5}$ (reject)

$a = 36$

$V_H = 36i + 15j$ ✓

(d) Calculate the resultant speed of the Helicopter.

$|V_r| = |9i + 36i + 15j|$ ✓

$= \sqrt{45^2 + 15^2}$

$= 47.434$ Km/hr ✓

(e) Find, ~~to the nearest minute~~, the time taken for the journey.

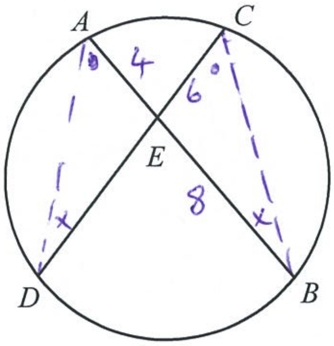
$|AB| = \sqrt{300^2 + 100^2} = 316.228$ Km ✓

$\text{Time} = \frac{316.228}{47.434} = 6.666 \text{ hrs} = 6 \text{hrs } 40 \text{ min}$ ✓

10

Question 5 [5 marks]

In the circle shown below, not to scale, *AB* and *CD* are chords that intersect at E. If *AE* = 4 cm, *BE* = 8 cm and *CE* = 6 cm, determine the length of *DE*. Justify your answer.



in ΔAED and ΔCEB

$\angle ADE = \angle CBE$ (angles on common chord)

$\angle DAE = \angle BCE$ (" " " ")

$\angle AED = \angle CEB$ (vertically opposite)

$\therefore \Delta AED \sim \Delta CEB$ (AAA)

$\therefore \dfrac{AE}{CE} = \dfrac{DE}{BE}$

$\dfrac{4}{6} = \dfrac{DE}{8}$

$DE = \dfrac{4 \times 8}{6}$

$= \dfrac{16}{3}$ or $5\frac{1}{3}$ cm

5

Question 6 [5 marks]

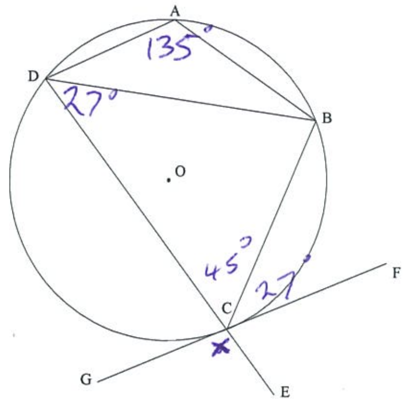
A, B, C and D are points on the circumference of a circle centre O.

GF is a tangent to the circle at C.

DE and *GF* intersect at C.

Angles *DAB* and *BDC* are $135°$ and $27°$ respectively.

Prove $\angle GCE = 72°$.



$\angle DCB = 45°$ $(45 + 135 = 180$, cyclic quadrilateral$)$

$\angle BCF = 27°$ (angle in alternate segment)

$\therefore \angle DCF = 45 + 27 = 72°$

$\therefore \angle GCE = \angle DCF$ (vertically opposite)

$= 72°$

for reasons.

5